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Tugas 4 Matematika 2

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1). $\int \sqrt[3]{\cos x}\ \sin x\, dx$

misal : $u = \cos x$

$du = -\sin x\, dx$

$= -\int u^{1/3}\, du$

$= -\frac{3}{4} u\sqrt{u} + C$

$= -\frac{3}{4} \cos x \sqrt{\cos x} + C$

//

2). $\int_0^1 \frac{x}{x^4+1}\, dx$

$= \int \frac{x}{(x^2)^2+1}\, dx$

misal : $u = x^2$ BA : $u = 1$

$du = 2x\, dx$ BB : $u = 0$

$= \frac{1}{2} \int_0^1 \frac{\cancel{x}}{u^2+1} \frac{du}{\cancel{x}}$

$= \frac{1}{2} \int_0^1 \frac{du}{u^2+1}$

$= \frac{1}{2} \left[\tan^{-1} u\right]_0^1$

$= \frac{1}{2} \left[\frac{\pi}{4} - 0\right]$

$= \frac{\pi}{8}$ //

3). $\int \frac{dx}{x^4 - 3x^3 - 7x^2 + 27x - 18}$

$$\begin{array}{c|ccccc} & 1 & -3 & -7 & 27 & -18 \\ 1 & & 1 & -2 & -9 & 18 \\ \hline & 1 & -2 & -9 & 18 & 0 \end{array}$$

$x^3 - 2x^2 - 9x + 18 = 0$

$$\begin{array}{c|cccc} & 1 & -2 & -9 & 18 \\ 2 & & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$(x^2 - 9) = 0$

$x^4 - 3x^3 - 7x^2 + 27x - 18 = (x-1)(x-2)(x+3)(x-3)$

$\int \frac{dx}{x^4-3x^3-7x^2+27x-18} = \int \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)} + \frac{D}{(x-3)} dx$

$1 = A(x-2)(x+3)(x-3) + B(x-1)(x+3)(x-3) + C(x-1)(x-2)(x-3) + D(x-1)(x-2)(x+3)$

→ Substitusi $x = 1$

$8A = 1$

$A = \frac{1}{8}$

→ Substitusi $x = 2$

$-5B = 1$

$B = -1/5$

→ Substitusi $x = -3$

$-120\ C = 1$

$C = -\frac{1}{120}$

→ Substitusi $x = 3$

$12D = 1$

$D = 1/12$

→ $\int \frac{1/8}{(x-1)} + \frac{(-1/5)}{(x-2)} + \frac{(-1/120)}{(x+3)} + \frac{1/12}{(x-3)}\, dx$

$= \frac{1}{8} \ln|x-1| - \frac{1}{5} \ln|x-2| - \frac{1}{120} \ln|x+3| + \frac{1}{12} \ln|x-3| + C$

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4). $\int e^t \sqrt{1-e^{2t}}\, dt$

misal : $u = e^t$

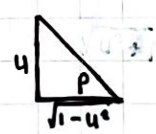
$du = e^t\, dt$

$\int \cancel{e^t} \sqrt{1-u^2} \cdot \frac{du}{\cancel{e^t}}$

$= \int \sqrt{1-u^2}\, du$

→ misal : $u = \sin p$ $du = \cos p\, dp$

$p = \sin^{-1} u$



$\cos p = \sqrt{1-u^2}$

→ $\int \sqrt{1-\sin^2 p} \cdot \cos p\, dp$

$= \int \sqrt{\cos^2 p}\ \cos p\, dp$

$= \int \cos^2 p\, dp$

$= \int \frac{\cos 2p + 1}{2}\, dp$

$= \int \frac{\cos 2p}{2} dp + \int \frac{1}{2}\, dp$

$= \frac{1}{2} \cdot \frac{1}{2} \sin 2p + \frac{1}{2} p + C$

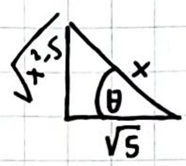
$= \frac{1}{4} \cdot 2 \sin p \cos p + \frac{1}{2} \sin^{-1} u + C$

$= \frac{1}{2} \sin(\sin^{-1} u) \cos(\sin^{-1} u) + \frac{1}{2} \sin^{-1} u + C$

$= \frac{1}{2} \left[u\sqrt{1-u^2} + \sin^{-1} u \right] + C$

$= \frac{1}{2} \left[e^t \sqrt{1-e^{2t}} + \sin^{-1}(e^t) \right] + C$

5). $\int \frac{\sqrt{x^2-5}}{x^4} \, dx$



misal : $x = \sqrt{5} \sec\theta$

$\sec\theta = \frac{x}{\sqrt{5}} \longrightarrow \sec\theta \tan\theta \, d\theta = \frac{dx}{\sqrt{5}}$

$dx = \sqrt{5} \sec\theta \tan\theta \, d\theta$

$\rightarrow \int \frac{\sqrt{(\sqrt{5}\sec\theta)^2 - 5}}{(\sqrt{5}\sec\theta)^4} \cdot \sec\theta \tan\theta \, d\theta \cdot \sqrt{5}$

$= \int \frac{\sqrt{5(\sec^2\theta - 1)}}{25 \sec^4\theta} \cdot \sqrt{5} \sec\theta \tan\theta \, d\theta$

$= \frac{5}{25} \int \frac{\tan\theta}{\sec^4\theta} \cdot \sec\theta \tan\theta \, d\theta$

$= \frac{1}{5} \int \frac{\tan^2\theta}{\sec^3\theta} \, d\theta$

$= \frac{1}{5} \int \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^3\theta \, d\theta$

$= \frac{1}{5} \int \sin^2\theta \cos\theta \, d\theta$

misal : $u = \sin\theta$

$du = \cos\theta \, d\theta$

$= \frac{1}{5} \int u^2 \, du$

$= \frac{1}{5} \cdot \frac{1}{3} u^3 + C$

$= \frac{1}{8} [\sin\theta]^3 + C$

$= \frac{1}{8} \cdot \left[\frac{\sqrt{x^2-5}}{x} \right]^3 + C$

$= \frac{1}{8} \cdot \frac{(x^2-5)\sqrt{x^2-5}}{x^3} + C$